2017 Montana Floodplain Resource Seminar: Lecture 2 Open Channel Hydraulics and Modeling

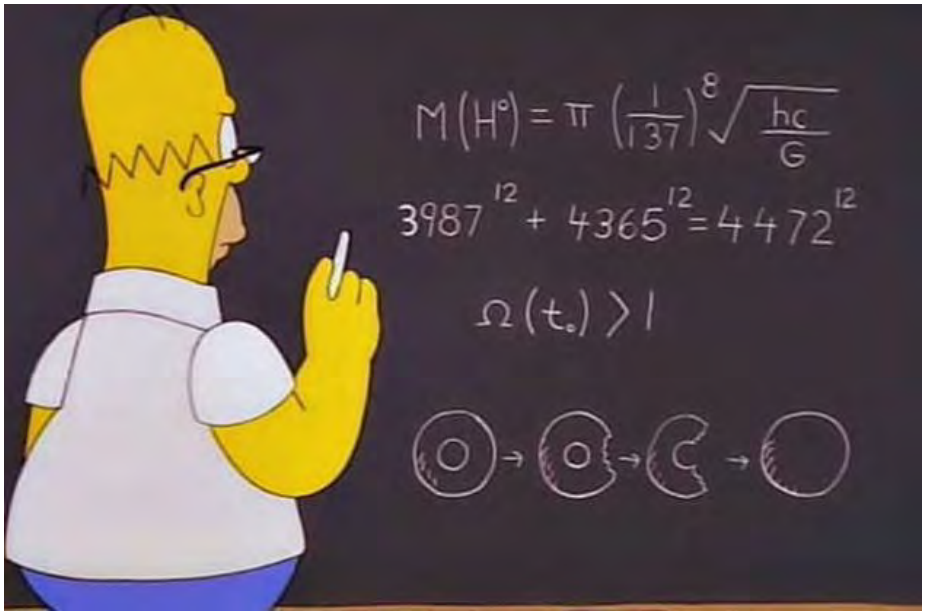
Eli Gruber, PE

compass

Identify Hazards, Interpret Risks, Integrate Mitigation

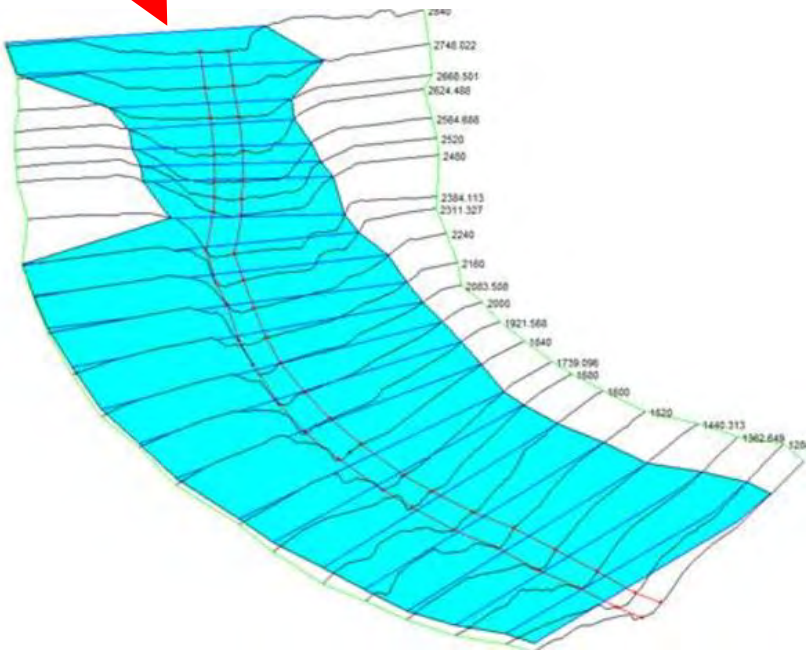
Lecture 2 Overview

- High level physical context
- Governing equations
- Steady flow
- Unsteady flow
- Numerical methods
- HEC-RAS 2D computations



compass
Identify, Interpret, Integrate

Why Do We Care?



compass
Identify, Interpret, Integrate

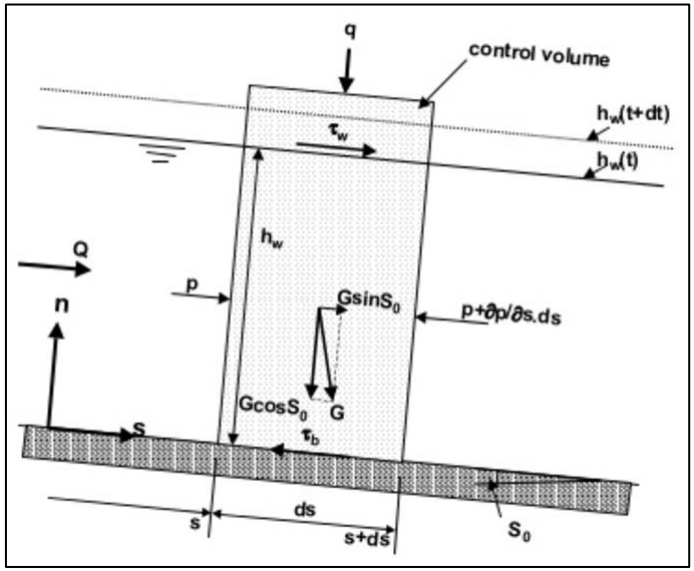
Fundamental Principles

- Complex interaction of water, solid surfaces and external forces
- "System"
  - For hydraulic modeling, the "system" is the water being represented
- Defined by physical properties
  - Mass and Weight
  - Velocity
  - Acceleration
  - Momentum
  - Mechanical Energy
- Basic principles of physics govern these properties and can be used to quantify and predict movement of water

compass
Identify, Interpret, Integrate

Physical Context

- Basic equations of fluid motion follow three step philosophy
  - Fundamental physical principles:
    - Energy is conserved
    - Mass is conserved
    - F=ma, Newton's 2nd law (Momentum)
  - Select suitable representation for "system"
    - Steady flow (1D) or unsteady flow (1D or 2D)
  - Extract mathematical equation(s) to define physical principles of selected representation



compass
Identify, Interpret, Integrate

1D Steady Flow

Flow does not change in space or time

Energy Equation:

$$Z_2 + Y_2 + \frac{a_2 V_2^2}{2g} = Z_1 + Y_1 + \frac{a_1 V_1^2}{2g} + h_e$$

Where: Z_1, Z_2 = elevation of the main channel inverts

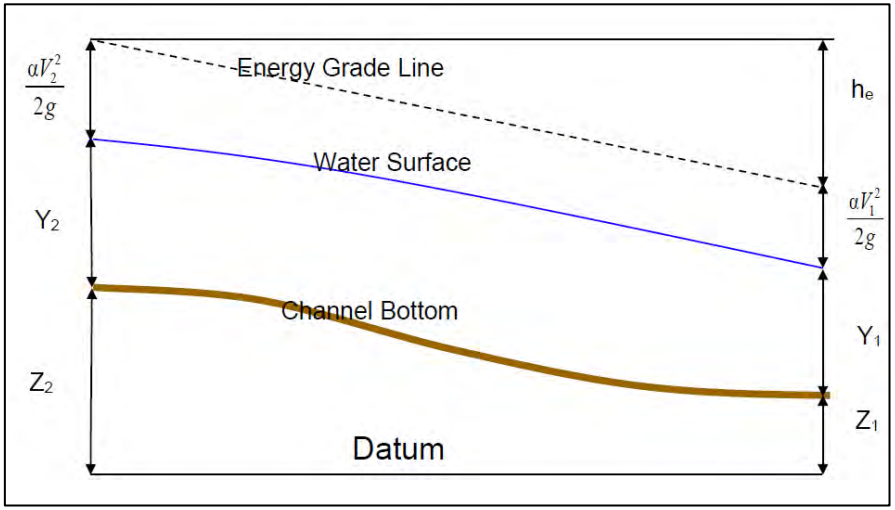
Y_1, Y_2 = depth of water at cross sections

V_1, V_2 = average velocities (total discharge/ total flow area)

a_1, a_2 = velocity weighting coefficients

g = gravitational acceleration

h_e = energy head loss



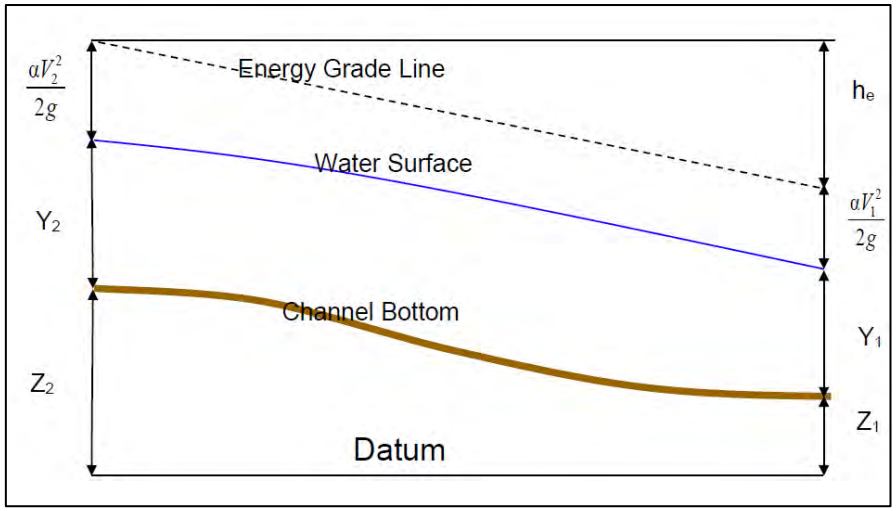
Start at known boundary condition, step upstream (subcritical) or downstream (supercritical) – Standard Step Method

compass
Identify, Interpret, Integrate

1D Energy Losses

Head loss defined as:

$$h_e = L\bar{S}_f + C\left|\frac{a_2V_2^2}{2g} - \frac{a_1V_1^2}{2g}\right|$$



- Friction losses
  - Manning's equation; empirical not physical

$$S_f = \left(\frac{Q}{K}\right)^2 \quad \text{where} \quad K = \frac{1.486}{n}AR^{2/3}$$

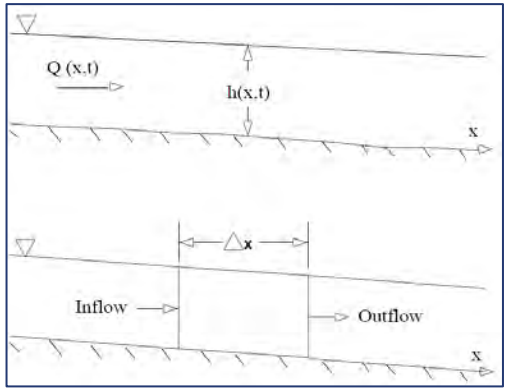
- Expansion and contraction losses

1D Unsteady Flow

Flow changes with respect to *x* and *t*

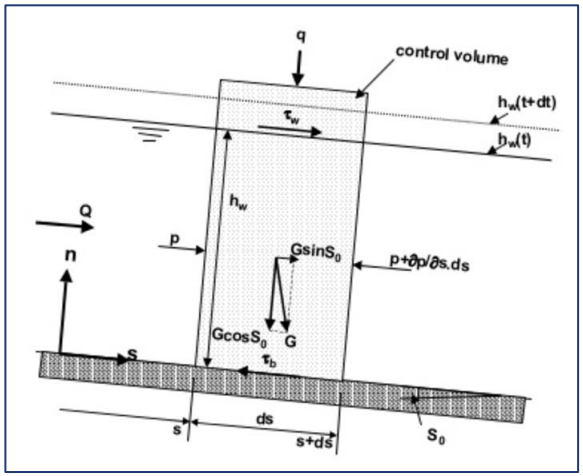
Continuity equation:

$$\frac{\partial A_T}{\partial t} + \frac{\partial Q}{\partial x} - q_l = 0$$



Momentum Equation:

$$\frac{\partial Q}{\partial t} + \frac{\partial QV}{\partial x} + gA\left(\frac{\partial z}{\partial x} + S_f\right) = 0$$



Need to determine *h* and *Q* at all sections for every timestep

compass
Identify, Interpret, Integrate

1D Unsteady Numerical Solution

- Continuity and Momentum equations are non-linear
- Based on a series of assumptions, equations can be linearized and solved with finite difference scheme
  - Finite difference methods discretize system into series of small pieces that can be solved numerically

1D Unsteady Numerical Scheme

- For a system of 2N nodes
  - 2N unknowns
  - 2N-2 equations are available
  - User defined upstream and downstream boundary conditions for the two unknowns w/out equation
- A matrix is developed of all linearized equations in the system that can be solved simultaneously
  - Allows for information from the entire reach to influence solution at any one point

compass
Identify, Interpret, Integrate

2D Unsteady Flow

Continuity

$$\frac{\partial H}{\partial t}+\frac{\partial(hu)}{\partial x}+\frac{\partial(hv)}{\partial y}+q=0$$

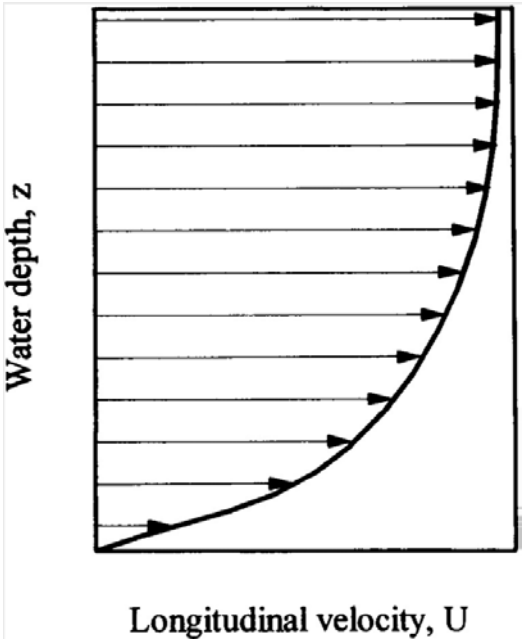
Momentum

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-g\frac{\partial H}{\partial x}+\nu_t\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-c_f u+fv$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-g\frac{\partial H}{\partial y}+\nu_t\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-c_f v+fu$$

u and v are velocities in the cartesian directions

Depth-averaged velocity assumed



compass
Identify, Interpret, Integrate

Diffusion Wave Approximation

- In shallow gravity and friction controlled flow, the full momentum equations can be approximated

Diffusion Wave Equation

$$V = \frac{-(R(H))^{2/3}}{n} \frac{\nabla H}{|\nabla H|^{1/2}}$$

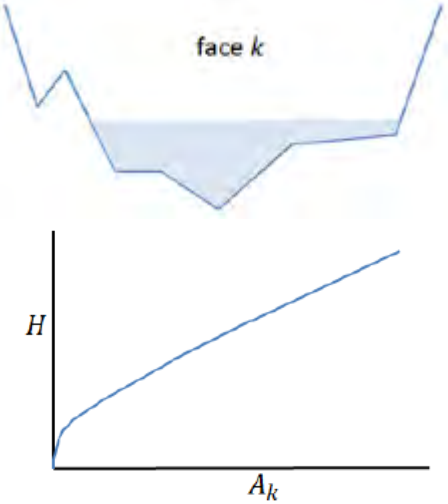
V is the velocity vector
R is the hydraulic radius
∇H is the surface elevation gradient

- This can be substituted into the Continuity Equation creating a 1-equation model
- Significantly more simple, but is not appropriate for many flow situations ~ **USE WITH CAUTION**

compass
Identify, Interpret, Integrate

2D Numerical Approach

- Combination of implicit finite difference (structured mesh) and implicit finite volume (unstructured mesh) methods
- Subgrid bathymetry considered by developing a series of property tables
  - Area-Elevation (face)
  - Wetted Perimeter-Elevation (face)
  - Manning's n-Elevation (face)
  - Profile (face)
  - Volume-Elevation (cell)



compass
Identify, Interpret, Integrate

Review

- Don’t need to be an expert in fluid dynamics or numerical methods, but must understand the concepts to be able to:
  - Make correct application
  - Understand model assumptions and limitations
  - Identify errors and model instability

compass
Identify, Interpret, Integrate

Questions

Photo from FEMA

compass

Identify Interpret Integrate