Appendix

Target Flow Rationale and Assumptions

1. Teton River above Springhill

- Flow rates and percentages were estimated based on monthly summed January- December 2017 flow data.
- Flow percentages and timing are not instantaneous values.
- Flow percentages for the Teton River at Saylor Bridge, were estimated using a regression equation and flow measurements made on Spring Creek.
- Gains from groundwater, not associated with management of the Springhill Reach (Groundwater flowing downgradient and losses from the Teton River above the Springhill Reach), will add additional flow to the Teton River near Choteau and Spring Creek.
- Target flows were developed using the above-mentioned percentages (36 and 30%) of flow and water demands on the Teton River above Choteau. The target flow rate was then reduced or increased based on current data (2017) and expected changing groundwater conditions.
- Flows during the months of April, May, June and July account for the Sexton and C Hanging L rights. The length and quantity of delivery of water for rights junior to the Broken O will vary year to year depending on priority and water supply.
- Flow diverted down the (TR 079) Stone Creek Ditch to below the Springhill gage was subtracted from inflow into the Springhill Reach.
- Careful monitoring will be needed to ensure that water rights are being met and there is not an excess of water generated.

2. Teton River at Dutton

- The target flow rate of 27 cfs is based on an estimate of 60% loss rate in the approximately 90 miles of river channel in the Lower River during the dry summer months.
- The flow rate is intended to keep the channel wet to near Loma and will facilitate the delivery and potential call of senior stock water rights.
- Travel time from Dutton to Loma during the summer is approximately 7-10 days when the channel is wet. If dry conditions exist it can but upwards of 12 days.
- Travel time from Choteau (Highway 221 gage) to Dutton during the summer is approximately 5-7 days.

Lower Teton River Travel Time Regression Equation

Manual measurements of (average velocity (ft/s) and discharge (ft^3/S)) from the following gages were collected over the 2008-2012 TWS study period: Teton River at Hwy 221, Teton River near Dutton (USGS), Teton River at Bootlegger Bridge, Teton River at Buck Bridge and Teton River at Loma (USGS). On any given day, the flow varies significantly along the Lower River. Given this fact, it was not deemed reasonable to create a regression equation based on measurements made on the same day.

The process to create the regression equation is as follows:

1. Flow and average velocity data from discharge measurements at individual sites were sorted from smallest discharge to largest discharge.
2. Velocity and discharge data from each site were categized into five cubic feet per second flow ranges (0-5, 5-10, etc.....).
3. Categorized data from all the gages were combined and averaged (e.g. all the 0-5 cfs data available for each gage was averaged into one velocity and one discharge value). This step was repeated for each category of data.
4. The average velocity and discharge values for each category were plotted and a polynomial trend line was fitted to the data.
5. Regression velocity estimates were checked against observed travel times to assess accuracy

The regression analysis assumes that:

- Flow conditions on the Lower River are stable.
- Non-flow conditions do not exist on the Lower River.

Flow data can be used from the following gages to estimate travel time in the appropriate reaches

- Teton River above Highway 221 (DNRC)
- Teton River near Dutton (USGS)
- Teton River at Buck Bridge (DNRC)
- Teton River at Loma (USGS)

The regression equation can be used to estimate the average velocity (ft/sec) of water in the Lower Teton River. Flow travel times are presented in Table 2 in the guidance document.

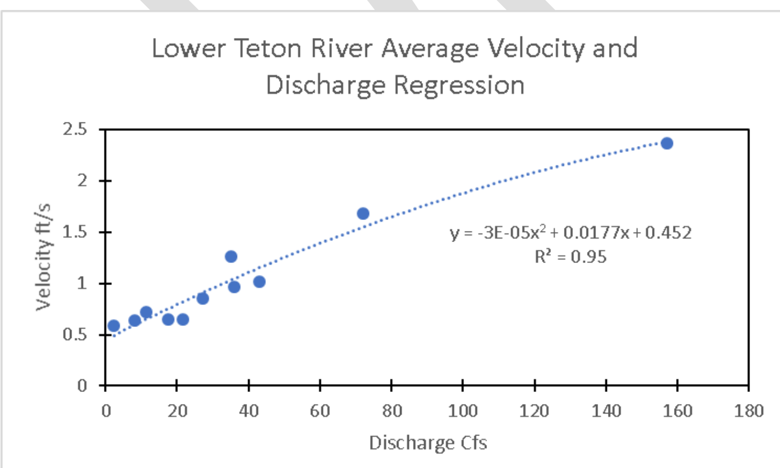


Figure 5: Regression chart and equation used to estimate velocities based on discharge for the Lower Teton River.

Springhill Reach Retention Time

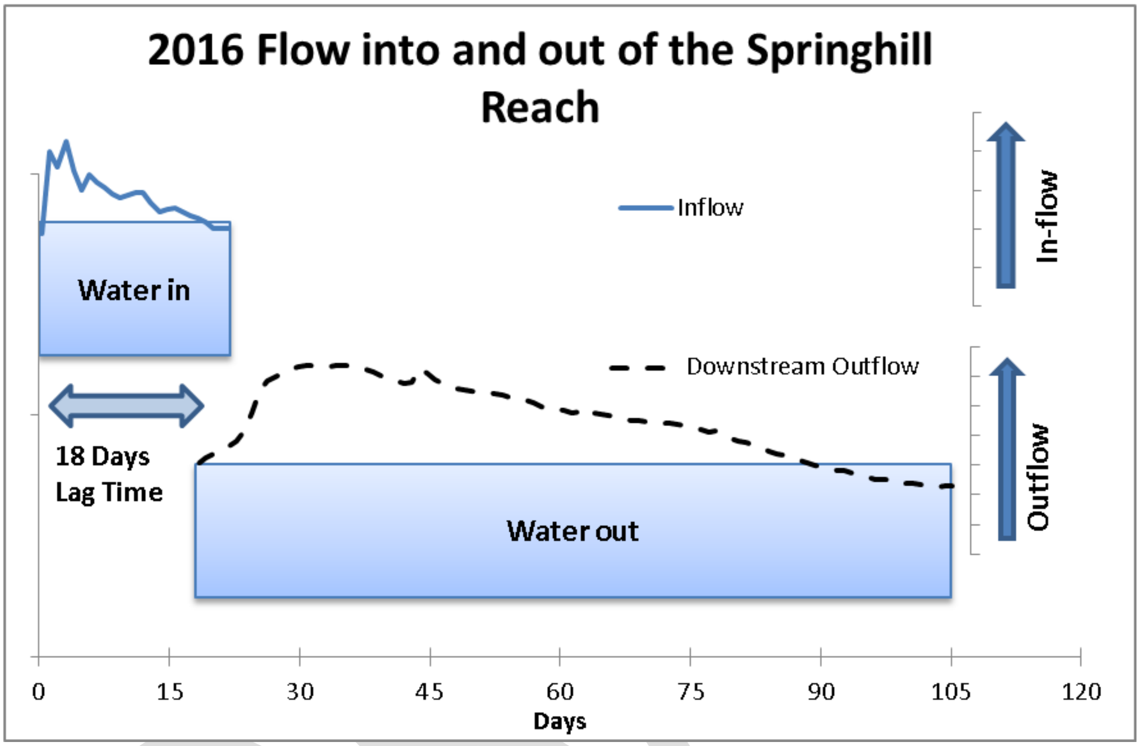


Figure 6: August 2016 streamflow of the Teton River water into the Springhill Reach and flow out of Upper Spring Creek

Estimation of Missing Discharge Records for the Teton River Below Bridge Near Saylor Lane

Introduction

Discharge measurements were made at three points in time (April 5, July 6, and August 31--2017) at the station located on the Teton River below the bridge near Saylor Lane; additional daily to weekly observations of stream stage were made occasionally at the site throughout the summer. A rating curve, developed from the discharge measurements, was used to estimate discharge from the stream stage observations, however the daily record estimated contained missing values due to missing stage observations.

In order to fill in the missing discharge records, a multiple, linear-regression analysis was used to establish a relationship between the estimated daily to weekly discharge (variable T_RatedQ) and potential explanatory variables. Several potential explanatory variables with daily observations were examined in the regression analysis (Table 1).

Table 1. Potential explanatory variables for filling in missing discharge observations

Time Series	Variable Name	Variable Type	Units
Madel Groundwater	Madel_GW_EL	Groundwater	Elevation in feet
Barhaugh Groundwater	Barh_GW_EL	Groundwater	Elevation in feet
South Fork Teton River Discharge	USGS_SFKQ	Streamflow	Mean daily discharge in cfs
Upper Spring Creek Discharge	UP_SPR_Q	Streamflow	Mean daily discharge in cfs
Teton River above Springhill Discharge	T_ABV_SPRH_Q	Streamflow	Mean daily discharge in cfs

Methods

Several least-squares, linear regression models (1.1) were fit to the data and examined to select the best-fit model that met the assumptions for least-squares, linear regression (Neter and others 1996).

(1.1)

$$
\begin{aligned}
&Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + ...\beta_{p-1} X_{p-1} + \varepsilon_i \\
&\text{where,} \\
&Y_i = \text{response variable,} \\
&\beta_0 = \text{intercept,} \\
&\beta_1 ... = \text{explanatory variables,} \\
&X_{i1}, ..., X_{i,p-1} \text{ are known constants,} \\
&\varepsilon_i = \text{are independent, normally distributed errors,} \\
&\quad \text{with mean zero and constant variance}
\end{aligned}
$$

Statistical assumptions for valid multiple-linear regression models are:

1. The mean of the response variable, at each set of values of the explanatory variables, is a linear function of the explanatory variables (i.e. the mean of the error, at each set of values of the explanatory variables, sum to zero);
2. The errors in the model fit are independent (i.e. are not correlated with each other);
3. The errors at each set of values of the explanatory variables, are normally distributed; and
4. The errors, at each set of values of the explanatory variables, have equal variances.

Another way of stating these assumptions is that the errors in the model are independent, normal, random variables with mean zero and constant variance. Errors are also the equivalent of the residuals (difference between predicted response and observed values) of the regression equation. These assumptions were tested through examination of residual scatterplots (i.e. plots of residuals against predicted values), normal probability plots, residual histograms, and statistical tests for normality of residuals.

Because the potential explanatory variables (Table 1) are located in the same area, and respond to the same stimuli (e.g. snowmelt runoff, seasonal groundwater recharge), they may be highly correlated with one another. This condition, referred to as multicollinearity, can lead to unstable model parameter estimates with high standard errors. Multicollinearity was diagnosed by use of the Variance Inflation Factor (VIF) and the examination of Condition Indices. If the VIF of an explanatory variable is greater than 5 (which means that the standard error for the coefficient of that explanatory variable is five times as large as it would be if the variable were uncorrelated with the other predictor variables), multicollinearity is a problem in the model. An additional diagnostic measure for detecting multicollinearity is Condition Indices (C.I.); values near 10 indicate weak dependencies may be affecting regression estimates—values greater than 100 indicate parameter estimates may have significant numerical error (Panik, 2009).

Finally, closely spaced observations (e.g. daily flow) may exhibit autocorrelation which inflates model parameter significance statistics by underestimating standard errors; autocorrelation was diagnosed using the Durbin-Watson D statistic. The Durbin-Watson statistic ranges from 0 to 4.0 and tests for first order autocorrelation in the error term (e.g. residuals). In general, a D statistic between 1.5 and 2.5 indicates the data are independent; a small value (less than 1.5) indicates positive first-order correlation and a large value (greater than 2.5) indicates negative first-order correlation (Panik 2009; Neter and others 1996).

Developing a multiple linear-regression model is an iterative process that consists of fitting various combinations of explanatory variables and examining regression diagnostics and model fit statistics to select the best model. All statistical analyses were performed using Statistical Analysis System (SAS) Software (v.9.4) and the REG, AUTOREG and UNIVARIATE procedures.

Results

<u>Model 1</u>

Regression Model 1 consists of the response variable, T_RatedQ, and all the potential explanatory variables given in Table 1. Model 1 results are summarized in Tables 1a and 1b (full results are given in Appendix 1).

Model 1 results (Table 1a.) show that the overall model is highly significant (Pr > F=<0.001) with an adjusted R-square of 0.7274 and model errors are normally distributed (Shapiro-Wilk Pr<W=0.9668). Examination of residual plots (Appendix 1.) indicate equal variance for each of the explanatory variables.
The Durbin-Watson D statistic suggests the presence of first-order positive autocorrelation which is to be expected of daily time-series data.

Model Significance (Pr > F)	Adjusted R-Square	Durbin-Watson D	Residuals Normality (Shapiro-Wilk Pr<W) p-value
<0.0001	0.7274	1.368	0.9668

Table 1a. Model 1 results: Model Significance, Adjusted R-Square, Durbin-Watson D, and Normality of Residuals

Model 1 results (Table 1b.) show that the intercept and three of the explanatory variables (Barh_GW_EL, USGS_SFKQ and UP_SPR_Q) are significant at the p=0.05 level; T_ABV_SPRH_Q is not and Madel_GW_EL is marginally significant (i.e. p-value of 0.0558 is close to p=0.05). The VIF's are in the acceptable range (e.g. <10), but the C.I.'s for UP_SPR_Q (78.7) and T_ABV_SPRH_Q (209.2) are large and indicate significant problems with multicollinearity.

Table 1b. Model 1 results: Parameter Significance, VIF and C.I.

Parameter	Parameter Significance (Pr>|t|) p-value	VIF	Condition Index (C.I.)
Intercept	<0.0001	0	1.00000
Madel_GW_EL	0.0558	2.00406	3.51678
Barh_GW_EL	<0.0001	1.94862	5.03098
USGS_SFKQ	<0.0001	3.91099	19.48881
UP_SPR_Q	<0.0001	4.92259	78.73136
T_ABV_SPRH_Q	0.5535	1.28680	209.15785

Model 2

Regression Model 2 consists of the response variable, T_RatedQ, and all the potential explanatory variables that were statistically (or nearly) significant in Model 1 (Madel_GW_EL, Barh_GW_EL, USGS_SFKQ and UP_SPR_Q). Model 2 results are summarized in Tables 2a. and 2b (full results are given in Appendix 1).

Model 2 results (Table 2a.) show that the overall model is highly significant (Pr > F=<0.001) with an adjusted R-square of 0.7320 and model errors are normally distributed (Shapiro-Wilk Pr<W=0.9573). Examination of residual plots (Appendix 1.) indicate equal variance for each of the explanatory variables—except perhaps for USGS_SFKQ at higher discharges. The Durbin-Watson D statistic suggests the presence of first-order positive autocorrelation like Model 1.

Table 2a. Model 2 results: Model Significance, Adjusted R-Square, Durbin-Watson D, and Normality of Residuals

Model Significance (Pr > F)	Adjusted R-Square	Durbin-Watson D	Residuals Normality (Shapiro-Wilk Pr<W) p-value
<0.0001	0.7320	1.334	0.9573

Model 2 results (Table 2b.) show that the intercept and three of the explanatory variables (Barh_GW_EL, USGS_SFKQ and UP_SPR_Q) are significant at the p=0.001 level; Madel_GW_EL is borderline. The VIF's are in the acceptable range (e.g. <10), but the C.I.'s for USGS_SFKQ (72.8) and UP_SPR_Q (193.8) and are large and indicate significant problems with multicollinearity.

Table 2b. Model 1 results: Parameter Significance, VIF and C.I.

Parameter	Parameter Significance (Pr>|t|) p-value	VIF	Condition Index (C.I.)
Intercept	<.0001	0	1.00000
Madel_GW_EL	0.0576	1.99642	3.47338
Barh_GW_EL	<.0001	1.94135	15.39878
USGS_SFKQ	<.0001	3.58923	72.78225
UP_SPR_Q	<.0001	4.32357	193.78355

Model 3

Regression Model 3 consists of the response variable, T_RatedQ, and all the potential explanatory variables that were statistically (or nearly) significant in Model 2 (Barh_GW_EL, USGS_SFKQ and UP_SPR_Q). In addition, because residual plots indicate seasonality in the data, time-harmonic terms were added for sine and cosine. Model 3 results are summarized in Tables 3a. and 3b (full results are given in Appendix 1).

Model 3 results (Table 3a.) show that the overall model is highly significant (Pr > F=<0.001) with an adjusted R-square of 0.8900 and model errors are normally distributed (Shapiro-Wilk Pr<W=0.8662). Examination of residual plots (Appendix 1.) indicate equal variance for each of the explanatory variables—except perhaps for USGS_SFKQ at higher discharges. The Durbin-Watson D statistic suggests the lack of first-order positive autocorrelation—possibly because of the addition of harmonic terms.

Table 3a. Model 3 results: Model Significance, Adjusted R-Square, Durbin-Watson D, and Normality of Residuals

Model Significance (Pr > F)	Adjusted R-Square	Durbin-Watson D	Residuals Normality (Shapiro-Wilk Pr<W) p-value
<0.0001	0.8900	1.499	0.8662

Model 3 results (Table 3b.) show that the intercept, COS2, Madel_GW_EL and UP_SPR_Q) are significant at the p=0.001 level-- Barh_GW_EL and USGS_SFKQ are not. The VIF's are less than 10 for Madel_GW_EL, Barh_GW_EL, and USGS_SFKQ but the remaining explanatory variables exceed 10; the C.I.'s for USGS_SFKQ (246.5) and UP_SPR_Q (306.1) and are large and indicate significant problems with multicollinearity.

Table 3b. Model 1 results: Parameter Significance, VIF and C.I.

Parameter	Parameter Significance (Pr>|t|) p-value	VIF	Condition Index (C.I.)
Intercept	<.0001	0	1.00000
SIN2	0.6743	12.36501	2.79745
COS2	<.0001	18.43300	7.60873
Madel_GW_EL	<.0001	7.35855	9.24016
Barh_GW_EL	0.3627	8.96496	39.38237
USGS_SFKQ	0.3095	7.54911	246.50009
UP_SPR_Q	<.0001	29.40997	306.11621

Model 4

Explanatory variables for Model 4 were selected based on the information generated by the previos model results; the model consists of the response variable, T_RatedQ, and SIN2, COS2, USGS_SFKQ and UP_SPR_Q, as explanatory variables (note that if one of the harmonic terms is significant—both terms are included). Model 4 results are summarized in Tables 4a. and 4b (full results are given in Appendix 1).

Model 4 results (Table 4a.) show that the overall model is highly significant (Pr > F=<0.001) with an adjusted R-square of 0.7321 and model errors are approximately normally distributed (Shapiro-Wilk Pr<W=0.0559). Examination of residual plots (Appendix 1.) indicate equal variance for each of the explanatory variables—except perhaps for USGS_SFKQ at higher discharges.

The Durbin-Watson D statistic suggests the presence of first-order positive autocorrelation similar to Models 1 and 2.

Table 4a. Model 4 results: Model Significance, Adjusted R-Square, Durbin-Watson D, and Normality of Residuals

Model Significance (Pr > F)	Adjusted R-Square	Durbin-Watson D	Residuals Normality (Shapiro-Wilk Pr<W) p-value
<0.0001	0.8019	1.234	0.8662

Model 4 results (Table 4b.) show that the intercept, SIN2, COS2, UP_SPR_Q, and USGS_SFKQ) are significant at the p=0.05 level. The VIF's are less than 10 for all variables except USGS_SFKQ (10.9); the C.I.'s for all variables do not exceed 35 and indicate weak dependencies that may affect regression estimates.

Table 4b. Model 1 results: Parameter Significance, VIF and C.I.

Parameter	Parameter Significance (Pr>|t|) p-value	VIF	Condition Index (C.I.)
Intercept	0.0329	0	1.0000
SIN2	0.0002	5.41652	2.33388
COS2	<0.0001	2.94792	6.27579
USGS_SFKQ	<0.0001	10.85008	9.01312
UP_SPR_Q	0.0002	2.77024	35.28567

The resulting multiple-regression equation is:

$$\text{T\_RatedQ} = -3.24941 + (-2.69934\,SIN2) + 8.16240\,COS2 + 2.67954\,\text{UP\_SPR\_Q} + (-0.00773\,\text{USGS\_SFQ})$$

where,

(1.2)

T_RatedQ = Teton River below bridgemean daily discharge (cfs),

$SIN2$ = time harmonic term,

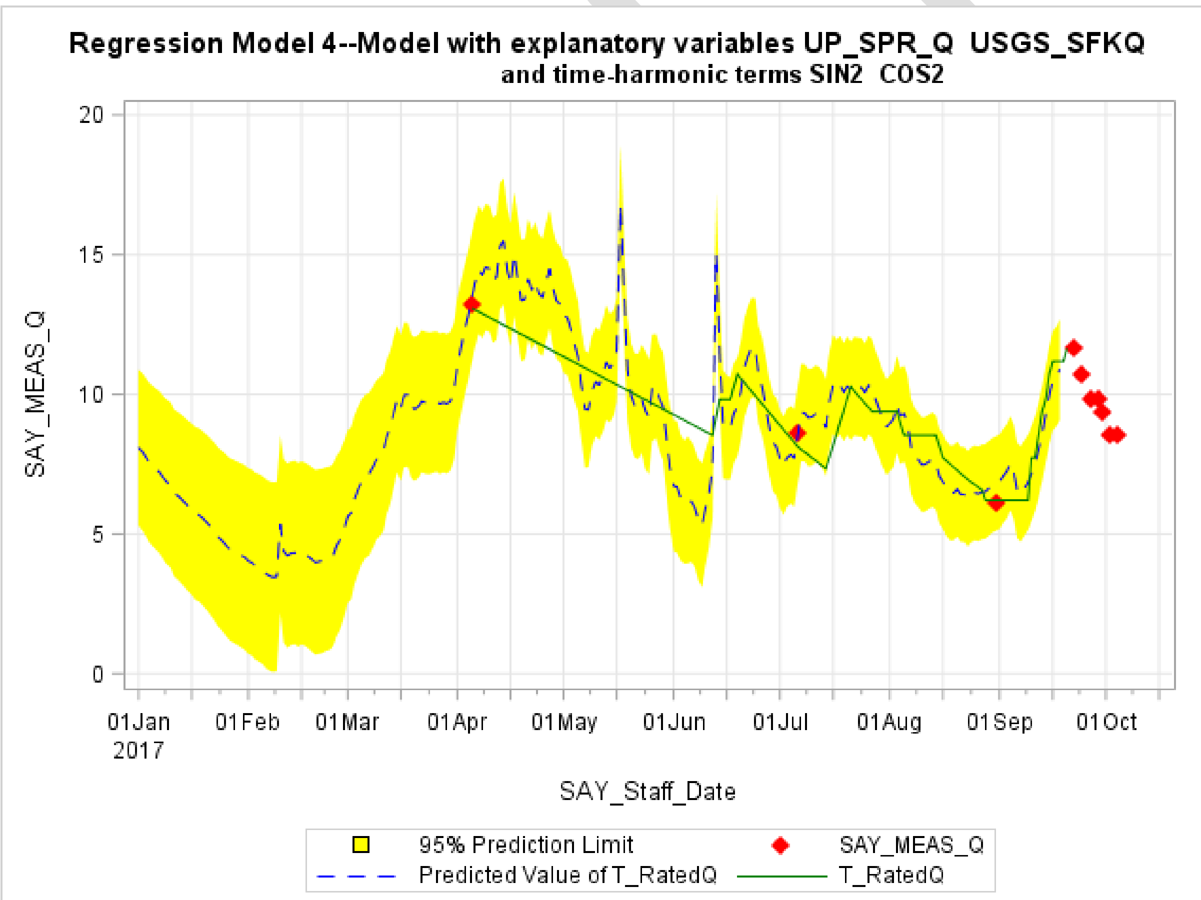
$COS2$ = ime harmonic term,

UP_SPR_Q = Upper Spring Creek mean daily discharge (cfs),

USGS_SFQ = South Fork Teton River, USGS mean daily discharge (cfs)

Model 4 results, including 95% prediction limits are shown in Figure 1.

Figure 1. Estimated missing daily discharge records for site on Teton River below Saylor Bridge



Model 5

All the previous models indicate that the residuals of the multiple-linear regression models are not independent (i.e. Durbin-Watson D statistics in the range of 1.3 to 1.5 indicating positive first order autocorrelation) but are autocorrelated, as is frequently the case in closely spaced time-series data such as daily streamflow. Autocorrelation violates a key assumption of linear ordinary least-squares regression and can result in under estimation of error variance, and lead to unreliable estimates of confidence intervals and hypothesis tests regarding significance of explanatory variables. The net effect is that the calculated t-statistics are inflated, thereby over estimating the statistical significance of explanatory variables, and the confidence intervals are narrower than they should be.

Accordingly, a regression model was fit that adjusts for the effect of autocorrelation. The model fit using PROC AUTOREG is:

$$\begin{aligned} \text{T\_RatedQ} = & -47.0395 + (1.643\text{SIN2}) + (9.703\text{COS2}) + (5.7559 Madel\_GW\_el) + (0.7589 UP\_SPR\_Q_t) \\ & + (-0.6823\varepsilon_{t-2}) + (0.1923\varepsilon_{t-5}) \end{aligned}$$

(1.3)

$$\begin{aligned} & where \\ & \text{T\_RatedQ}_t = Teton\ River\ below\ bridge\ near\ Saylor\ Lane\ (cfs)), \\ & SIN2_t = \text{time harmonic term}, \\ & COS2_t = \text{time harmonic term}, \\ & \text{Madel\_GW\_el}_t = \text{Groundwater elevation (feet)}, \\ & \text{UP\_SPR\_Q}_t = \text{Upper Spring Creek mean daily discharge (cfs)}, \\ & and \quad \varepsilon_t = -\zeta_{t-1+\upsilon_t}, \varsigma = -\rho \end{aligned}$$

The procedure simultaneously estimates regression coefficients of the explanatory variables and ρ.

A final model model, with explanatory variables Madel_GW_el, UP_SPR_Q, and time-harmonic terms SIN2 and COS2, provided the best predictor of T_Rated Q (Appendix1). An initial model with five lags was fit and the best model, with two AR terms (AR2 and AR5), was selected using backstep elimination. The final model's Durbin-Watson statistics were not significant (p=0.05) at any of the lags showing the effects of autocorrelation has been effectively modeled. The transformed regression R-square (0.8157) represents the statistic for a regression of transformed variables adjusted for the estimated autocorrelation. The total R-squared (0.9345) is computed from the autoregressive model residuals and reflects the better fit provided using lagged residuals.

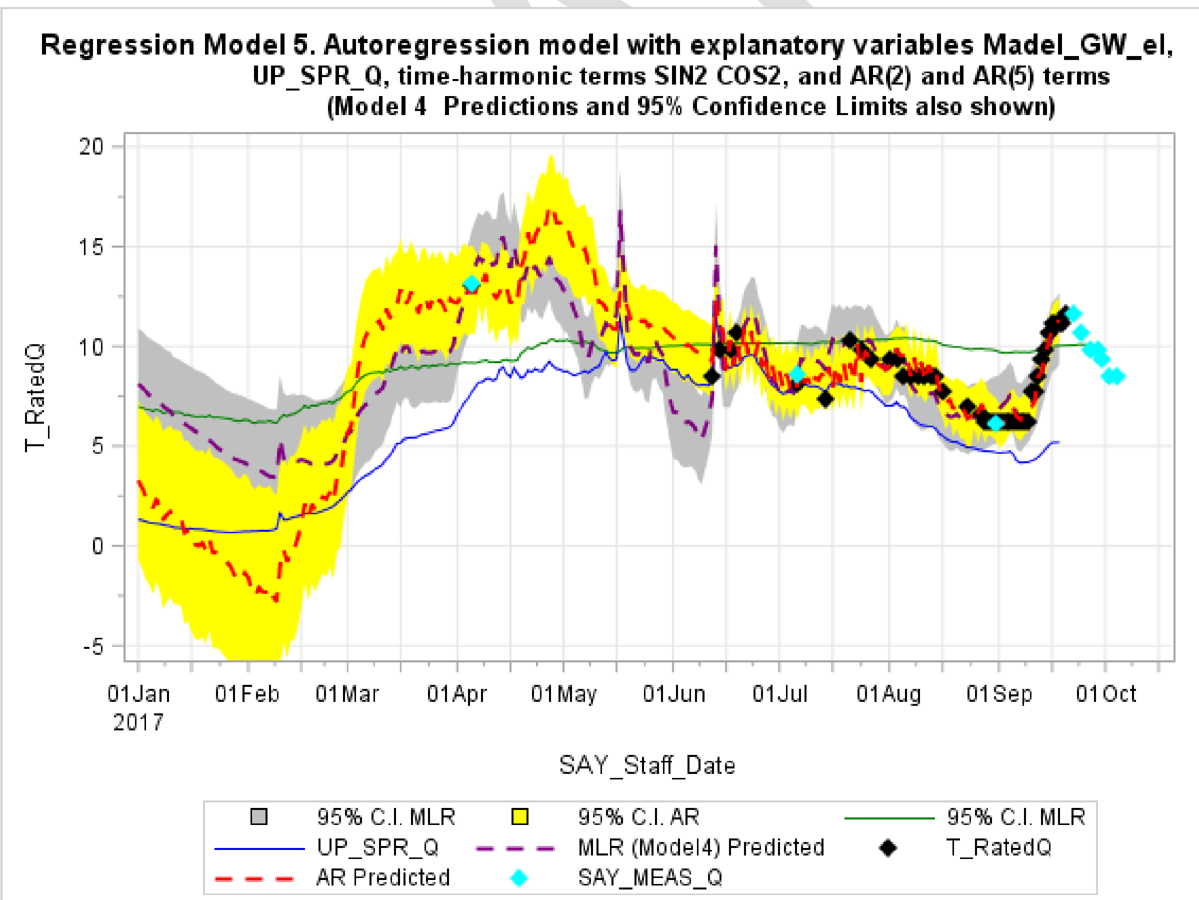
Normality of residuals is indicated by p-values for all the tests (p >0.05) and the box plot and Q/Q plot. Estimated model parameters and significance are given in Table 5; explanatory variables, the autoregression model predicted values along with 95% confidence intervals are shown ion Figure 2. An Excel file (Saylor_Model5_Results1.xslx) is provided with the results and explanatory variables.

Model predictions fit the available observations of T_RatedQ well (Figure 2). Negative discharges predicted by the model (latter part of January 2017 and February 2017) suggest that the stream may have dried (or frozen) up during the low-flow period.

Table 5. Regression Model 5. Autoregression model with explanatory variables Madel_GW_el, U P_SPR_Q, time-harmonic terms SIN2 COS2, and AR(2) and AR(5)

Statistic or Parameter	Estimated Value	Parameter Significance (Pr>|t|) p-value
Shapiro_Wilk (W) Normality of Residuals from Model 5	--	0.8848
Transformed Regression R-Square*	0.8157	
Total R-Square*	0.9345	
Intercept	-47.0395	<.0001
Madel_GW_el	5.7559	<.0001
SIN2	1.6413	0.0247
COS2	9.0703	<.0001
UP_SPR_Q	0.7589	0.0013
AR(2)	-0.6823	<.0001
AR(5)	0.1923	0.0932

Figure 2. Model 5—Best Regression Model



References:

Neter, J.; Kutner, M.H., Nachtsheim, C.J., and W. Wasserman 1996 *Applied Linear Statistical Models*. 4th Edition McGraw-Hill

Panik, M 2009 *Regression Modeling—Methods, Theory, and Computation with SAS.* CRC Press

APPENDIX 1. Saylor Regression Results

Regression Model 1--Full model with all potential explanatory variables
(Madel_GW_el Barh_GW_EL USGS_SFKQ UP_SPR_Q T_ABV_SPRH_Q)

The REG Procedure
Model: MODEL1
Dependent Variable: T_RatedQ

Number of Observations Read	274
Number of Observations Used	43
Number of Observations with Missing Values	231

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	107.41766	21.48353	23.42	<.0001
Error	37	33.94419	0.91741		
Corrected Total	42	141.36184			

Root MSE	0.95782	**R-Square**	0.7599
Dependent Mean	8.28884	**Adj R-Sq**	0.7274
Coeff Var	11.55549		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variance Inflation
Intercept	**1**	-53.18307	10.51008	-5.06	<.0001	0
Madel_GW_el	**1**	1.50236	0.76085	1.97	0.0558	2.00406
Barh_GW_EL	**1**	8.24332	1.07192	7.69	<.0001	1.94862
USGS_SFKQ	**1**	-0.01526	0.00261	-5.84	<.0001	3.91099
UP_SPR_Q	**1**	0.88510	0.20181	4.39	<.0001	4.92259
T_ABV_SPRH_Q	**1**	0.00545	0.00911	0.60	0.5535	1.28680

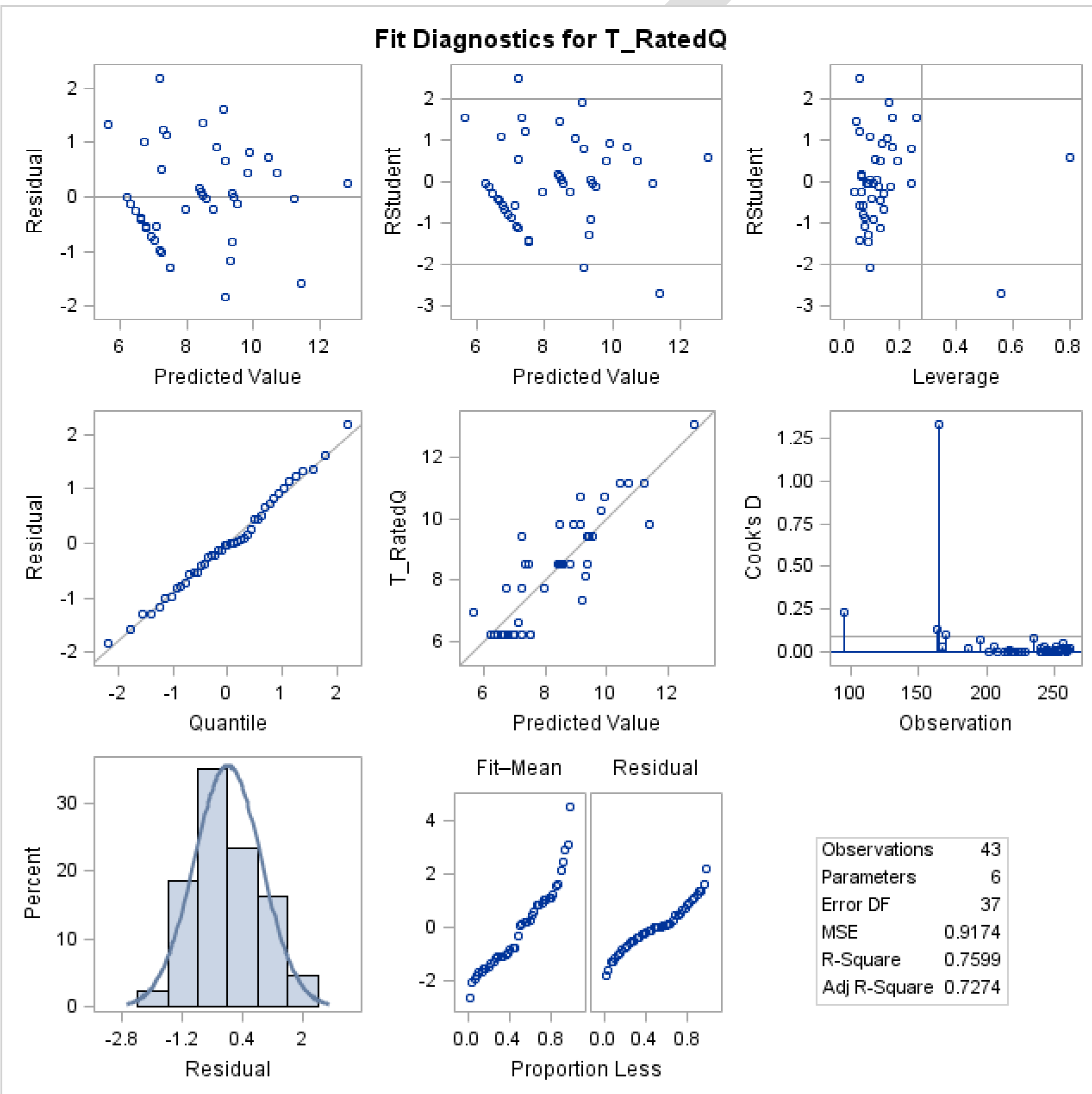
Collinearity Diagnostics

Number	Eigenvalue	Condition Index	Proportion of Variation					
			Intercept	Madel_GW_el	Barh_GW_EL	USGS_SFKQ	UP_SPR_Q	T_ABV_SPRH_Q
1	5.34197	1.00000	0.00000658	0.00001261	0.00002347	0.00282	0.00047628	0.00546
2	0.43193	3.51678	0.00001361	0.00002127	0.00003309	0.19404	0.00100	0.08893
3	0.21106	5.03098	0.00005672	0.00012140	0.00016459	0.06287	0.00443	0.60558
4	0.01406	19.48881	0.00108	0.00087376	0.00440	0.51778	0.71889	0.28983
5	0.00086180	78.73136	0.00395	0.18005	0.44160	0.20832	0.08219	0.00737
6	0.00012211	209.15785	0.99488	0.81892	0.55378	0.01417	0.19301	0.00283

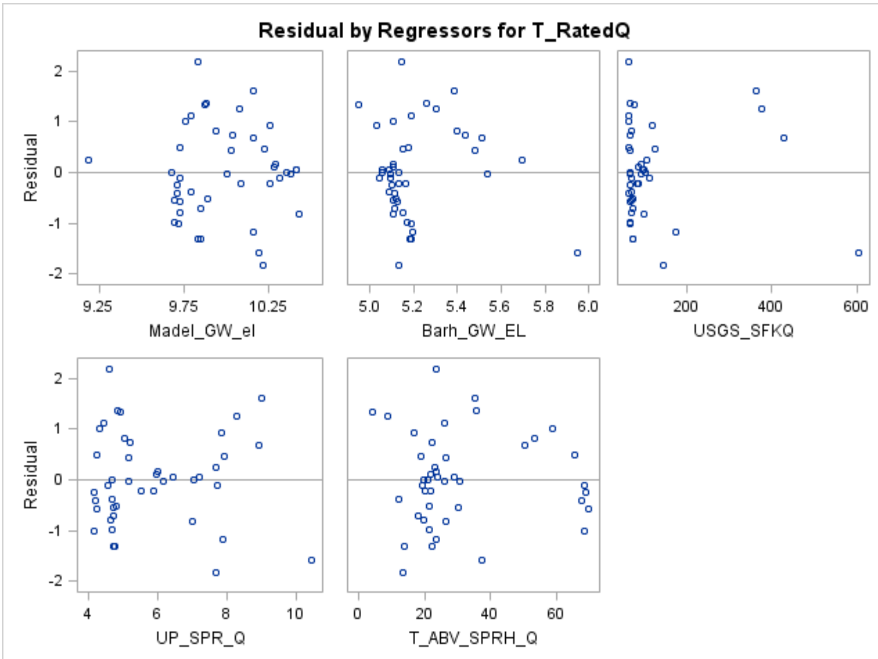
The REG Procedure
Model: MODEL1
Dependent Variable: T_RatedQ

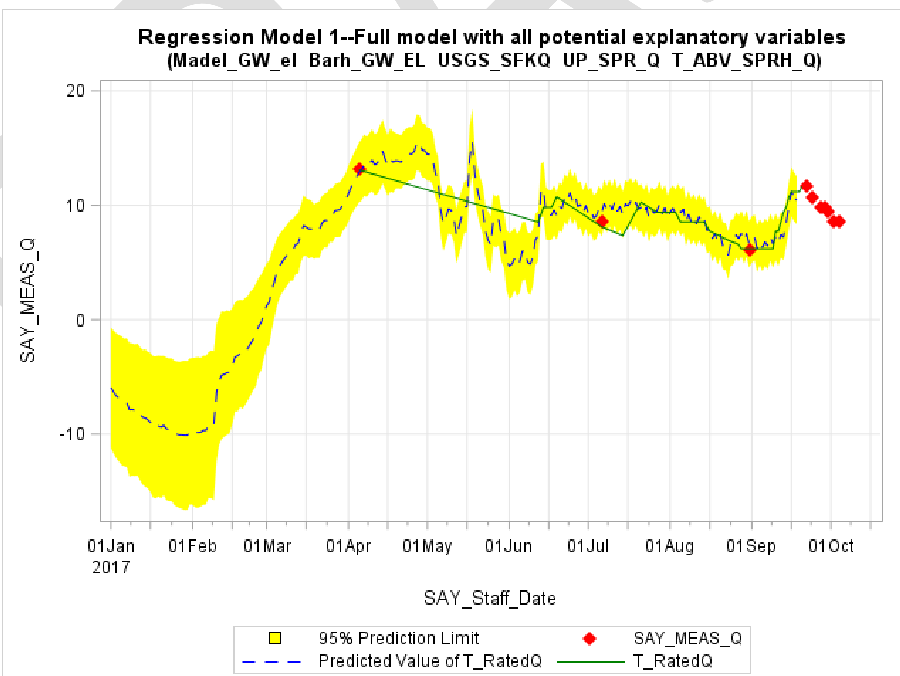
Durbin-Watson D	1.368
Number of Observations	43
1st Order Autocorrelation	0.307

The REG Procedure
Model: MODEL1
Dependent Variable: T_RatedQ



Regression Model 1--Full model with all potential explanatory variables
(Madel_GW_el Barh_GW_EL USGS_SFKQ UP_SPR_Q T_ABV_SPRH_Q)





Regression Model 1--Full model with all potential explanatory variables
(Madel_GW_el Barh_GW_EL USGS_SFKQ UP_SPR_Q T_ABV_SPRH_Q)

The UNIVARIATE Procedure
Variable: resid (Residual)

Moments

N	43	**Sum Weights**	43
Mean	0	**Sum Observations**	0
Std Deviation	0.89899662	**Variance**	0.80819492
Skewness	0.2009111	**Kurtosis**	-0.224608
Uncorrected SS	33.9441867	**Corrected SS**	33.9441867
Coeff Variation	.	**Std Error Mean**	0.1370957

Basic Statistical Measures

Location		**Variability**	
Mean	0.00000	**Std Deviation**	0.89900
Median	-0.02736	**Variance**	0.80819
Mode	.	**Range**	4.00545
		Interquartile Range	1.25496

Tests for Location: Mu0=0

Test	**Statistic**		**p Value**	
Student's t	**t**	0	**Pr > |t|**	1.0000
Sign	**M**	-2.5	**Pr >= |M|**	0.5424
Signed Rank	**S**	-12	**Pr >= |S|**	0.8868

Tests for Normality

Test	**Statistic**		**p Value**	
Shapiro-Wilk	**W**	0.990096	**Pr < W**	0.9688
Kolmogorov-Smirnov	**D**	0.083088	**Pr > D**	>0.1500
Cramer-von Mises	**W-Sq**	0.032744	**Pr > W-Sq**	>0.2500
Anderson-Darling	**A-Sq**	0.175675	**Pr > A-Sq**	>0.2500

Regression Model 2--Model with all Model 1 significant (p<0.05) explanatory variables (Madel_GW_el Barh_GW_EL USGS_SFKQ UP_SPR_Q)

The REG Procedure
Model: MODEL 2
Dependent Variable: T_RatedQ

Number of Observations Read	274
Number of Observations Used	43
Number of Observations with Missing Values	231

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	107.08969	26.77242	29.68	<.0001
Error	38	34.27216	0.90190		
Corrected Total	42	141.36184			

Root MSE	0.94968	**R-Square**	0.7576
Dependent Mean	8.28884	**Adj R-Sq**	0.7320
Coeff Var	11.45738		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variance Inflation
Intercept	**1**	-52.74354	10.39533	-5.07	<.0001	0
Madel_GW_el	**1**	1.47428	0.75295	1.96	0.0576	1.99642
Barh_GW_EL	**1**	8.28247	1.06083	7.81	<.0001	1.94135
USGS_SFKQ	**1**	-0.01481	0.00248	-5.97	<.0001	3.58923
UP_SPR_Q	**1**	0.84301	0.18752	4.50	<.0001	4.32357

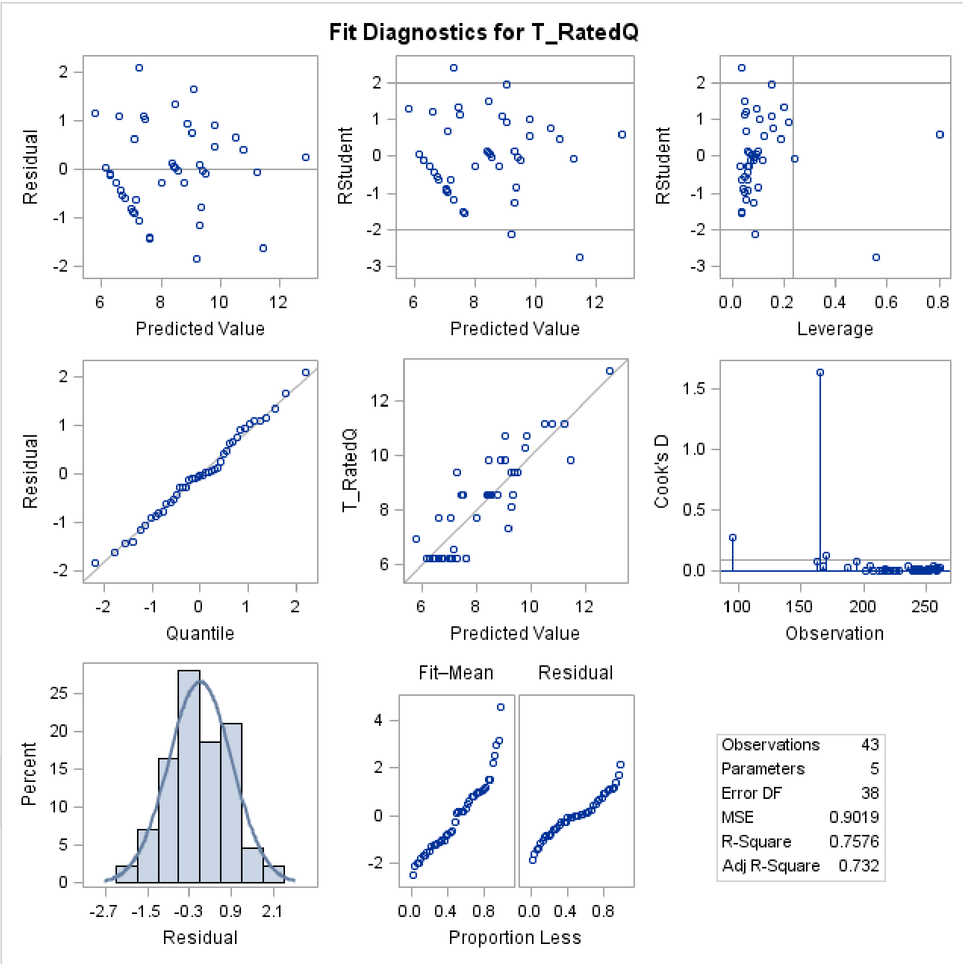
Collinearity Diagnostics

Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	**Madel_GW_el**	**Barh_GW_EL**	**USGS_SFKQ**	**UP_SPR_Q**
1	4.59846	1.00000	0.00000888	0.00001703	0.00003165	0.00433	0.00074766
2	0.38116	3.47338	0.00004181	0.00007284	0.00011328	0.26555	0.00015469
3	0.01939	15.39878	0.00061316	0.00036590	0.00271	0.46326	0.64173
4	0.00086808	72.78225	0.00410	0.18169	0.43409	0.25504	0.11808
5	0.00012246	193.78355	0.99523	0.81785	0.56306	0.01182	0.23928

Regression Model 2--Model with all Model 1 significant (p<0.05) explanatory variables (Madel_GW_el Barh_GW_EL USGS_SFKQ UP_SPR_Q)

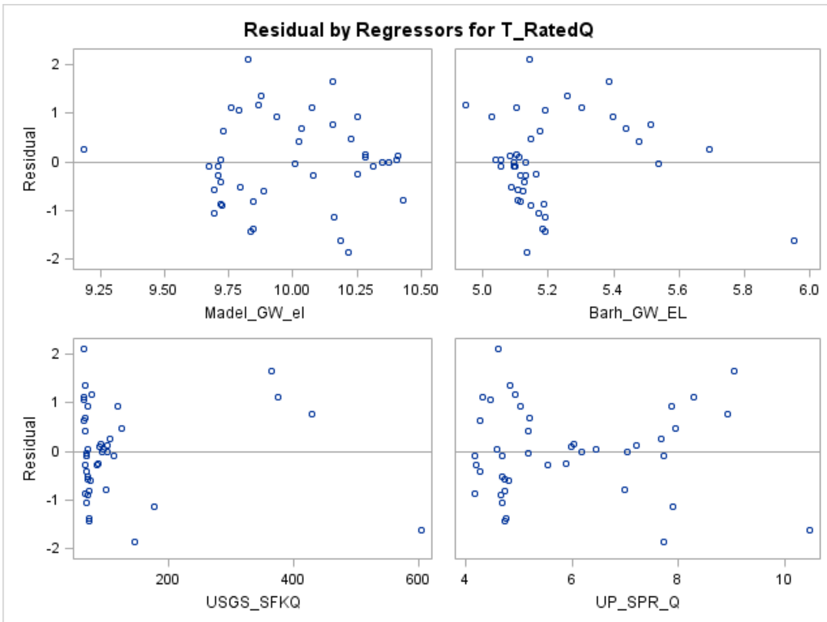
The REG Procedure
Model: MODEL 2
Dependent Variable: T_RatedQ

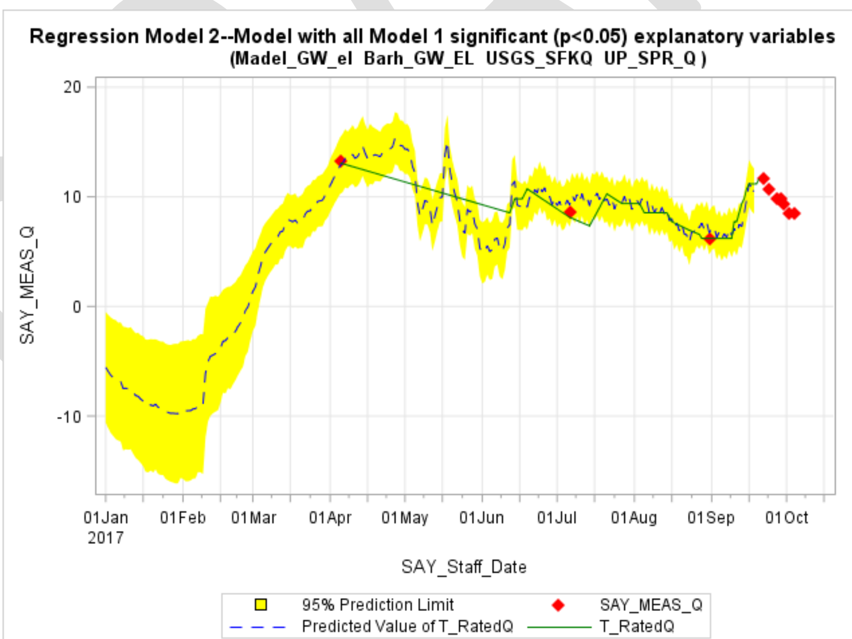
Durbin-Watson D	1.334
Number of Observations	43
1st Order Autocorrelation	0.325



Regression Model 2--Model with all Model 1 significant (p<0.05) explanatory variables
(Madel_GW_el Barh_GW_EL USGS_SFKQ UP_SPR_Q)

The REG Procedure
Model: MODEL2
Dependent Variable: T_RatedQ





Regression Model 2--Model with all Model 1 significant (p<0.05) explanatory variables
(Madel_GW_el Barh_GW_EL USGS_SFKQ UP_SPR_Q)

The UNIVARIATE Procedure
Variable: resid (Residual)

Moments

N	43	**Sum Weights**	43
Mean	0	**Sum Observations**	0
Std Deviation	0.90332923	**Variance**	0.81600371
Skewness	0.10083434	**Kurtosis**	-0.3096674
Uncorrected SS	34.2721556	**Corrected SS**	34.2721556
Coeff Variation	.	**Std Error Mean**	0.13775642

Basic Statistical Measures

Location		**Variability**	
Mean	0.00000	**Std Deviation**	0.90333
Median	-0.01578	**Variance**	0.81600
Mode	.	**Range**	3.95470
		Interquartile Range	1.27825

Tests for Location: Mu0=0

Test	**Statistic**		**p Value**	
Student's t	**t**	0	**Pr > |t|**	1.0000
Sign	**M**	-1.5	**Pr >= |M|**	0.7608
Signed Rank	**S**	-7	**Pr >= |S|**	0.9338

Tests for Normality

Test	**Statistic**		**p Value**	
Shapiro-Wilk	**W**	0.989399	**Pr < W**	0.9573
Kolmogorov-Smirnov	**D**	0.088327	**Pr > D**	>0.1500
Cramer-von Mises	**W-Sq**	0.034777	**Pr > W-Sq**	>0.2500
Anderson-Darling	**A-Sq**	0.189816	**Pr > A-Sq**	>0.2500

Regression Model 3--Model with explanatory variables Madel_GW_el Barh_GW_EL USGS_SFKQ UP_SPR_Q and time-harmonic terms SIN2 COS2

The REG Procedure
Model: MODEL 3
Dependent Variable: T_RatedQ

Number of Observations Read	274
Number of Observations Used	43
Number of Observations with Missing Values	231

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	128.02958	21.33826	57.62	<.0001
Error	36	13.33226	0.37034		
Corrected Total	42	141.36184			

Root MSE	0.60856	**R-Square**	0.9057
Dependent Mean	8.28884	**Adj R-Sq**	0.8900
Coeff Var	7.34188		

Parameter Estimates

Variable	Label	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variance Inflation
Intercept	Intercept	1	-38.96977	8.59347	-4.53	<.0001	0
SIN2	Sine*Theta	1	0.30900	0.72930	0.42	0.6743	12.36501
COS2	Cosine*Theta	1	11.55474	1.53965	7.50	<.0001	18.43300
Madel_GW_el		1	5.22551	0.92632	5.64	<.0001	7.35855
Barh_GW_EL		1	-1.34679	1.46080	-0.92	0.3627	8.96496
USGS_SFKQ		1	-0.00238	0.00231	-1.03	0.3095	7.54911
UP_SPR_Q		1	1.62700	0.31341	5.19	<.0001	29.40997

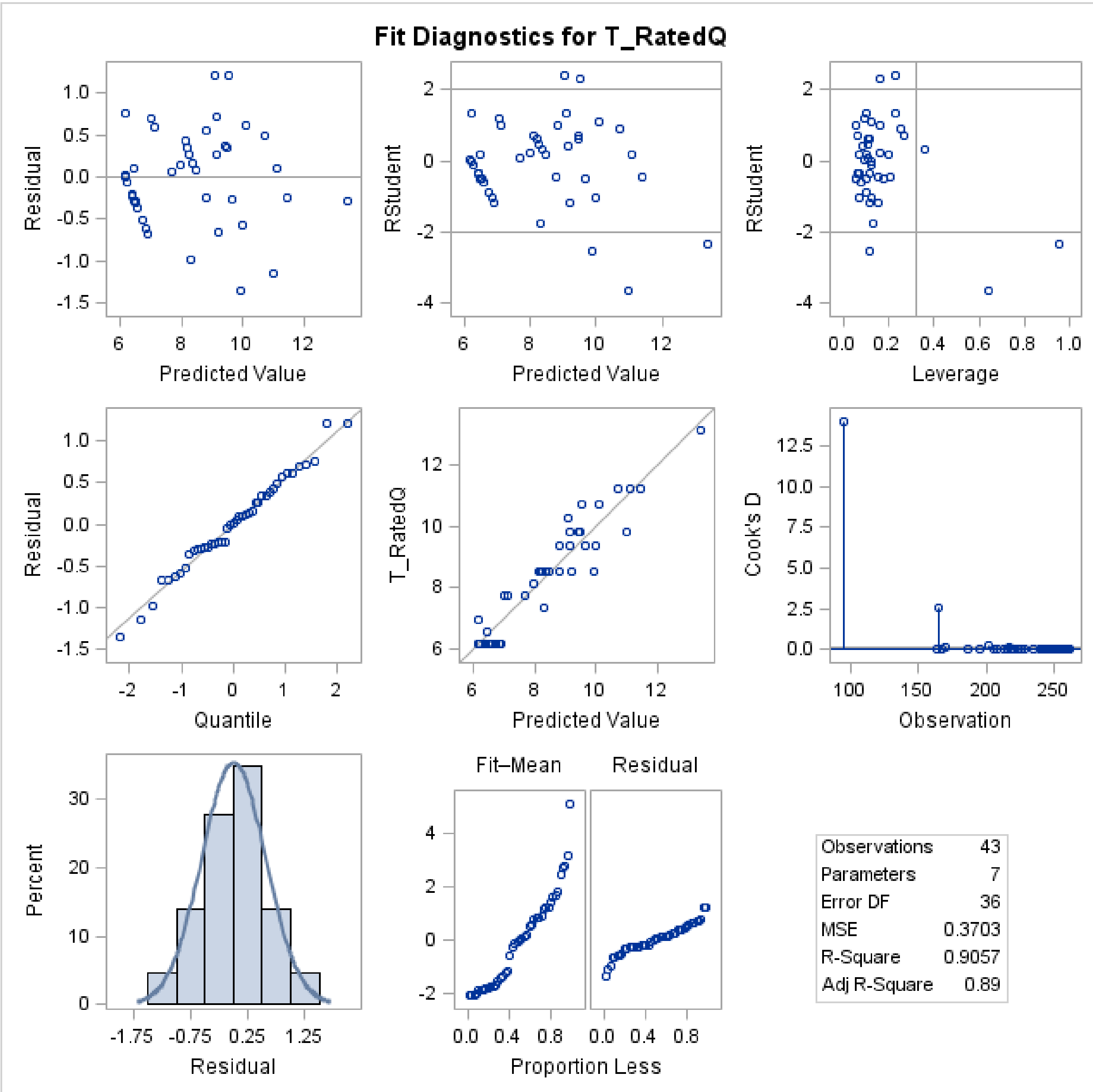
Collinearity Diagnostics

Number	Eigenvalue	Condition Index	Proportion of Variation						
			Intercept	SIN2	COS2	Madel_GW_el	Barh_GW_EL	USGS_SFKQ	UP_SPR_Q
1	6.04782	1.00000	0.00000313	0.00045112	0.00020949	0.00000272	0.00000400	0.00108	0.00006244
2	0.77281	2.79745	0.00000189	0.01863	0.00033622	0.00000135	0.00000151	0.03251	0.00008553
3	0.10447	7.60873	0.00000181	0.04896	0.02457	0.00000491	3.362184E-7	0.26763	0.00130
4	0.07083	9.24016	0.00015212	0.06901	0.05495	0.00008197	0.00028362	0.03071	0.00151
5	0.00390	39.38237	0.00431	0.34619	0.08703	0.00091325	0.00145	0.03108	0.41987
6	0.00009953	246.50009	0.12005	0.01342	0.83014	0.25261	0.94800	0.62439	0.06322
7	0.00006454	306.11621	0.87548	0.50334	0.00276	0.74639	0.05025	0.01261	0.51394

Regression Model 3--Model with explanatory variables Madel_GW_el Barh_GW_EL USGS_SFKQ UP_SPR_Q and time-harmonic terms SIN2 COS2

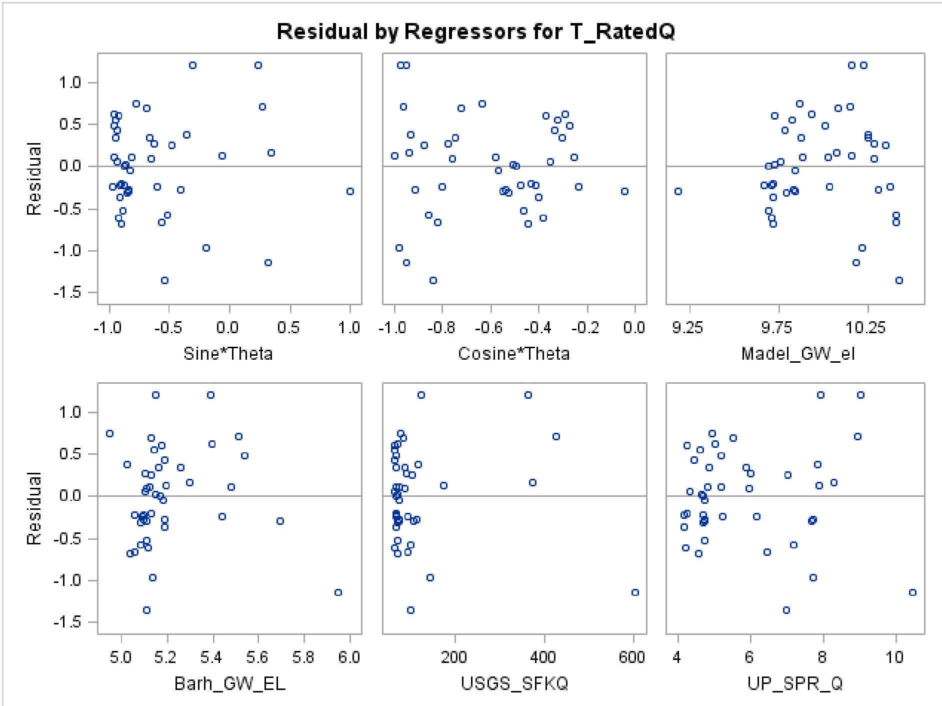
The REG Procedure
Model: MODEL 3
Dependent Variable: T_RatedQ

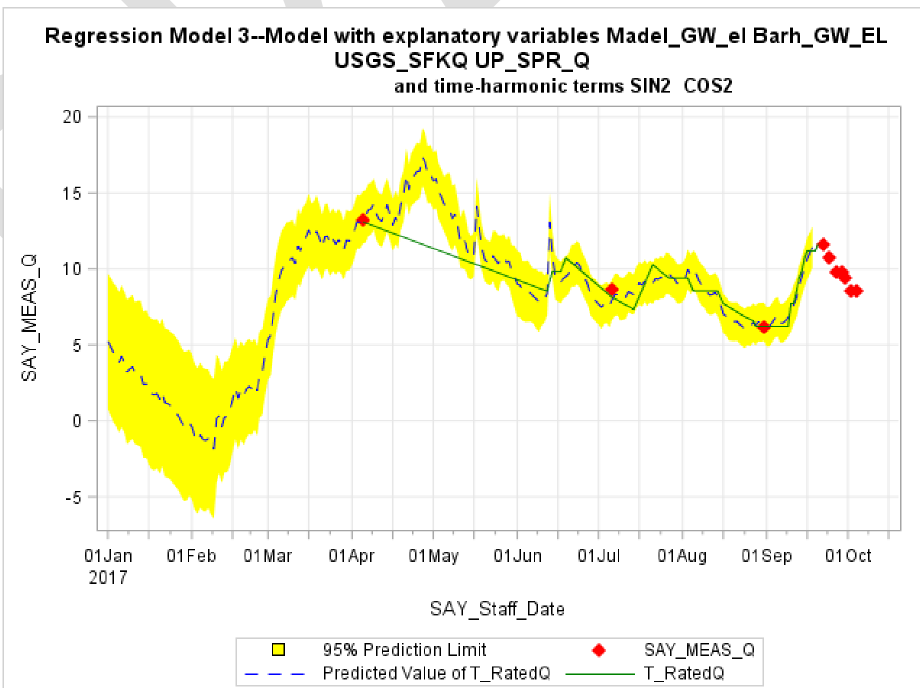
Durbin-Watson D	1.499
Number of Observations	43
1st Order Autocorrelation	0.245



Regression Model 3--Model with explanatory variables Madel_GW_el Barh_GW_EL USGS_SFKQ UP_SPR_Q and time-harmonic terms SIN2 COS2

The REG Procedure
Model: MODEL 3
Dependent Variable: T_RatedQ





Regression Model 3--Model with explanatory variables Madel_GW_el Barh_GW_EL USGS_SFKQ UP_SPR_Q and time-harmonic terms SIN2 COS2

The UNIVARIATE Procedure
Variable: resid (Residual)

Moments

N	43	**Sum Weights**	43
Mean	0	**Sum Observations**	0
Std Deviation	0.56341351	**Variance**	0.31743478
Skewness	-0.0964739	**Kurtosis**	0.15667447
Uncorrected SS	13.3322608	**Corrected SS**	13.3322608
Coeff Variation	.	**Std Error Mean**	0.08591975

Basic Statistical Measures

Location		**Variability**	
Mean	0.000000	**Std Deviation**	0.56341
Median	0.017060	**Variance**	0.31743
Mode	.	**Range**	2.56899
		Interquartile Range	0.66666

Tests for Location: Mu0=0

Test	**Statistic**		**p Value**	
Student's t	**t**	0	**Pr > |t|**	1.0000
Sign	**M**	1.5	**Pr >= |M|**	0.7608
Signed Rank	**S**	8	**Pr >= |S|**	0.9244

Tests for Normality

Test	**Statistic**		**p Value**	
Shapiro-Wilk	**W**	0.98584	**Pr < W**	0.8662
Kolmogorov-Smirnov	**D**	0.08905	**Pr > D**	>0.1500
Cramer-von Mises	**W-Sq**	0.034346	**Pr > W-Sq**	>0.2500
Anderson-Darling	**A-Sq**	0.228375	**Pr > A-Sq**	>0.2500

Regression Model 4--Model with explanatory variables UP_SPR_Q USGS_SFKQ
and time-harmonic terms SIN2 COS2

The REG Procedure
Model: MODEL 4
Dependent Variable: T_RatedQ

Number of Observations Read	274
Number of Observations Used	43
Number of Observations with Missing Values	231

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	116.02305	29.00576	43.50	<.0001
Error	38	25.33879	0.66681		
Corrected Total	42	141.36184			

Root MSE	0.81658	**R-Square**	0.8208
Dependent Mean	8.28884	**Adj R-Sq**	0.8019
Coeff Var	9.85162		

Parameter Estimates

Variable	Label	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variance Inflation
Intercept	Intercept	1	-3.24941	1.46783	-2.21	0.0329	0
SIN2	Sine*Theta	1	-2.69934	0.64769	-4.17	0.0002	5.41652
COS2	Cosine*Theta	1	8.16240	0.82619	9.88	<.0001	2.94792
UP_SPR_Q		1	2.67954	0.25544	10.49	<.0001	10.85088
USGS_SFKQ		1	-0.00773	0.00187	-4.12	0.0002	2.77024

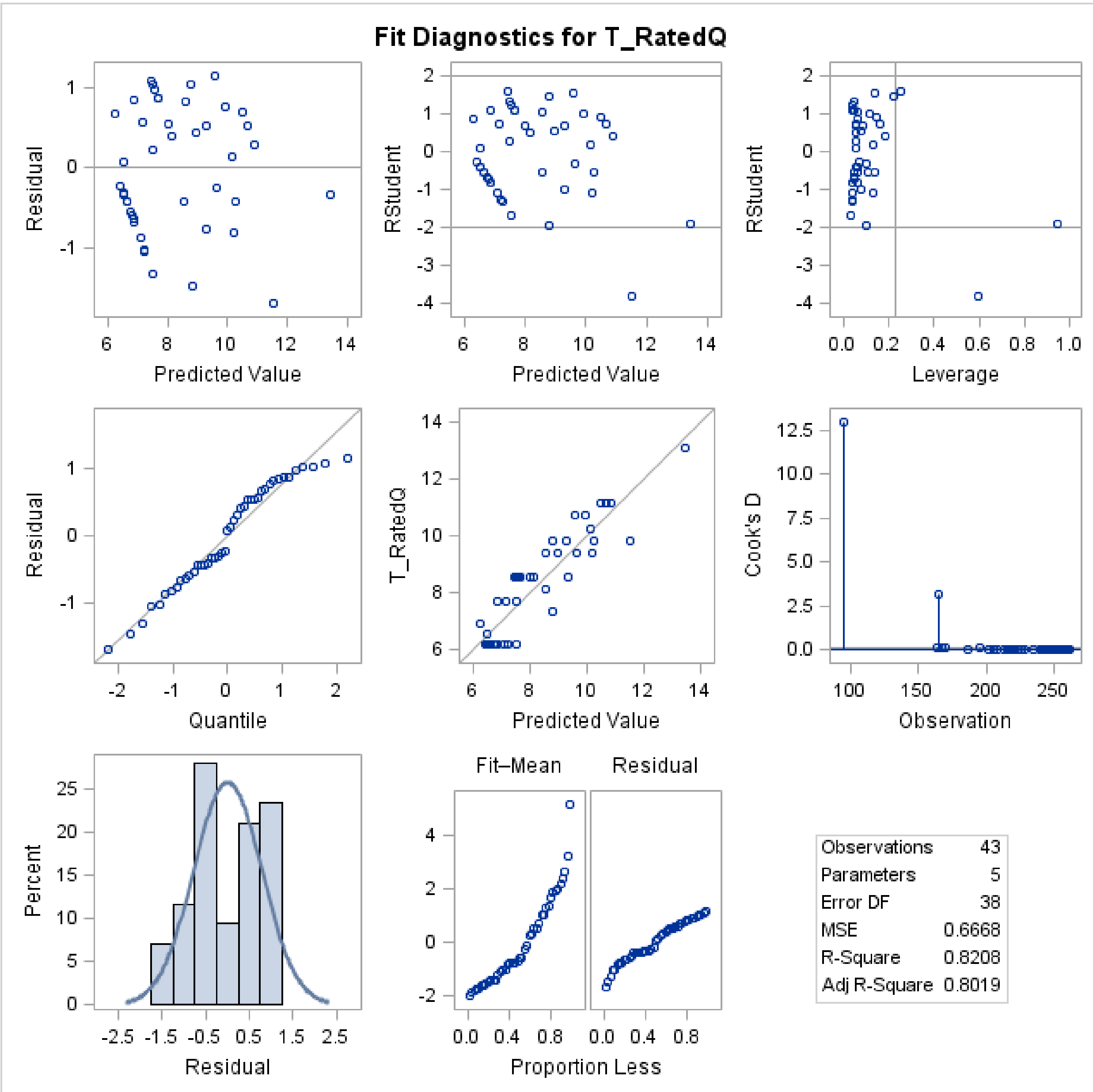
Collinearity Diagnostics

Number	Eigenvalue	Condition Index	Proportion of Variation				
			Intercept	SIN2	COS2	UP_SPR_Q	USGS_SFKQ
1	4.09135	1.00000	0.00041395	0.00209	0.00293	0.00037205	0.00690
2	0.75112	2.33388	0.00026277	0.05093	0.00099575	0.00011593	0.08028
3	0.10388	6.27579	0.00012725	0.10027	0.17069	0.00364	0.71819
4	0.05036	9.01312	0.04269	0.10805	0.41164	0.02431	0.10797
5	0.00329	35.28567	0.95650	0.73866	0.41374	0.97156	0.08666

Regression Model 4--Model with explanatory variables UP_SPR_Q USGS_SFKQ
and time-harmonic terms SIN2 COS2

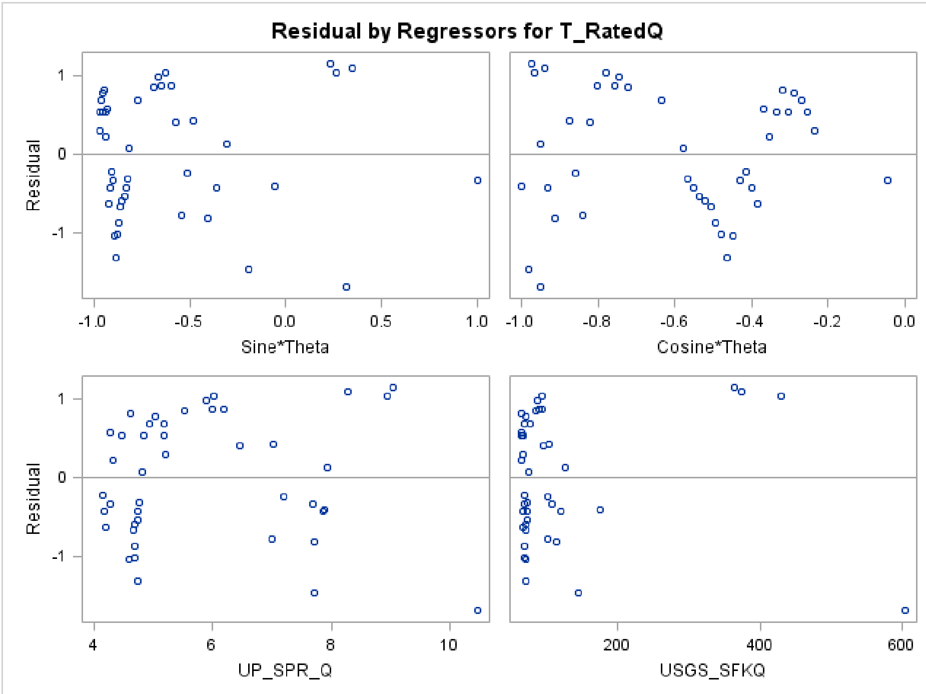
The REG Procedure
Model: MODEL 4
Dependent Variable: T_RatedQ

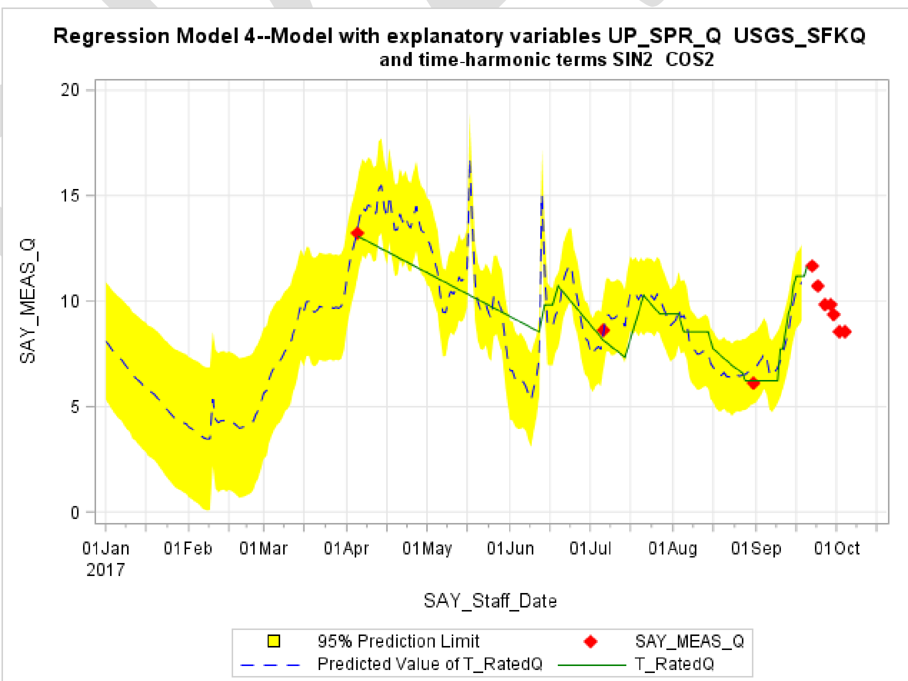
Durbin-Watson D	1.234
Number of Observations	43
1st Order Autocorrelation	0.379



Regression Model 4--Model with explanatory variables UP_SPR_Q USGS_SFKQ
and time-harmonic terms SIN2 COS2

The REG Procedure
Model: MODEL 4
Dependent Variable: T_RatedQ





Regression Model 5. Autoregression model with explanatory variables Madel_GW_el, UP_SPR_Q, time-harmonic terms SIN2 COS2, and AR(2) and AR(5)

The AUTOREG Procedure

Dependent Variable T_RatedQ

Ordinary Least Squares Estimates

SSE	16.0523076	**DFE**	38
MSE	0.42243	**Root MSE**	0.64995
SBC	98.4647719	**AIC**	89.6587713
MAE	0.45431711	**AICC**	91.2803929
MAPE	5.48204713	**HQC**	92.9061522
		Total R-Square	0.8864

Durbin-Watson Statistics

Order	DW	Pr < DW	Pr > DW
1	1.3824	0.0037	0.9963
2	2.1633	0.5848	0.4152
3	1.5981	0.0747	0.9253
4	1.7411	0.2294	0.7706
5	2.4833	0.9718	0.0282

NOTE: Pr<DW is the p-value for testing positive autocorrelation, and Pr>DW is the p-value for testing negative autocorrelation.

Parameter Estimates

Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|	Variable Label
Intercept	1	-50.6367	7.0550	-7.18	<.0001	
Madel_GW_el	1	5.9526	0.8519	6.99	<.0001	
SIN2	1	0.8893	0.7442	1.19	0.2395	Sine*Theta
COS2	1	10.6331	0.7606	13.98	<.0001	Cosine*Theta
UP_SPR_Q	1	1.1018	0.2512	4.39	<.0001	

Estimates of Autocorrelations

```
Lag  Covariance  Correlation  -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
0    0.3733      1.000000     |                    |********************|
1    0.0873      0.233943     |                    |*****               |
2    0.1045      0.279962     |                    |******              |
3    0.0883      0.236522     |                    |*****               |
4    0.00118     0.003157     |                    |                    |
5    -0.0875     -0.234477    |               *****|                    |
```

Backward Elimination of Autoregressive Terms

Lag	Estimate	t Value	Pr > |t|
4	0.083916	0.51	0.6103
1	-0.099185	-0.63	0.5357
3	-0.271999	-1.74	0.0901

Preliminary MSE	0.3083

Regression Model 5. Autoregression model with explanatory variables Madel_GW_el, UP_SPR_Q, time-harmonic terms SIN2 COS2, and AR(2) and AR(5)

Estimates of Autoregressive Parameters

Lag	Coefficient	Standard Error	t Value
2	-0.355297	0.155883	-2.28
5	0.318512	0.155883	2.04

Expected Autocorrelations

Lag	Autocorr
0	1.0000
1	-0.0846
2	0.4061
3	-0.1594
4	0.1712
5	-0.3751

Algorithm converged.

Maximum Likelihood Estimates

SSE	9.25846378	**DFE**	36
MSE	0.25718	**Root MSE**	0.50713
SBC	91.9294007	**AIC**	79.6009999
MAE	0.37604118	**AICC**	82.8009999
MAPE	4.68360463	**HQC**	84.1473331
Log Likelihood	-32.8005	**Transformed Regression R-Square**	0.8157
		Total R-Square	0.9345
		Observations	43

Durbin-Watson Statistics

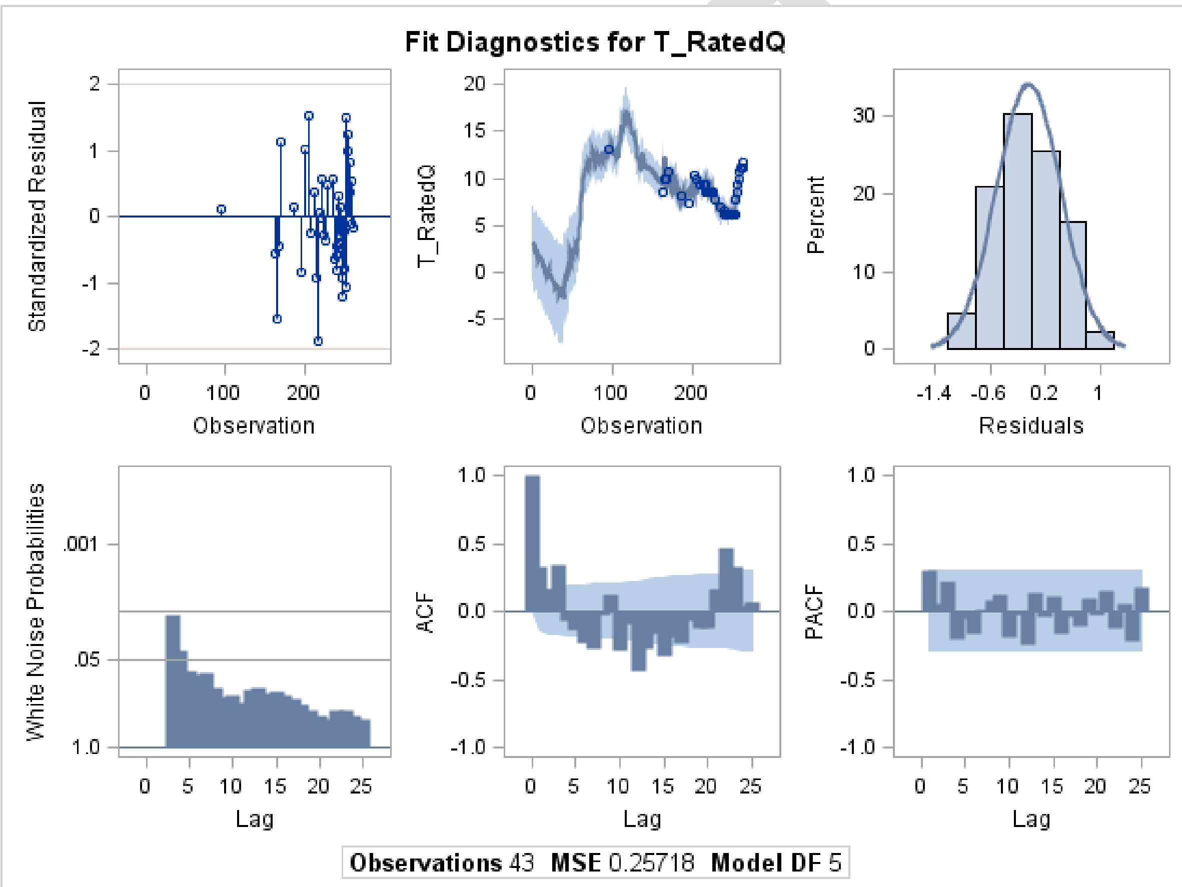
Order	DW	Pr < DW	Pr > DW
1	1.6796	0.0729	0.9271
2	2.9840	0.9995	0.0005
3	1.7460	0.2200	0.7800
4	1.3247	0.0218	0.9782
5	1.9522	0.6218	0.3782

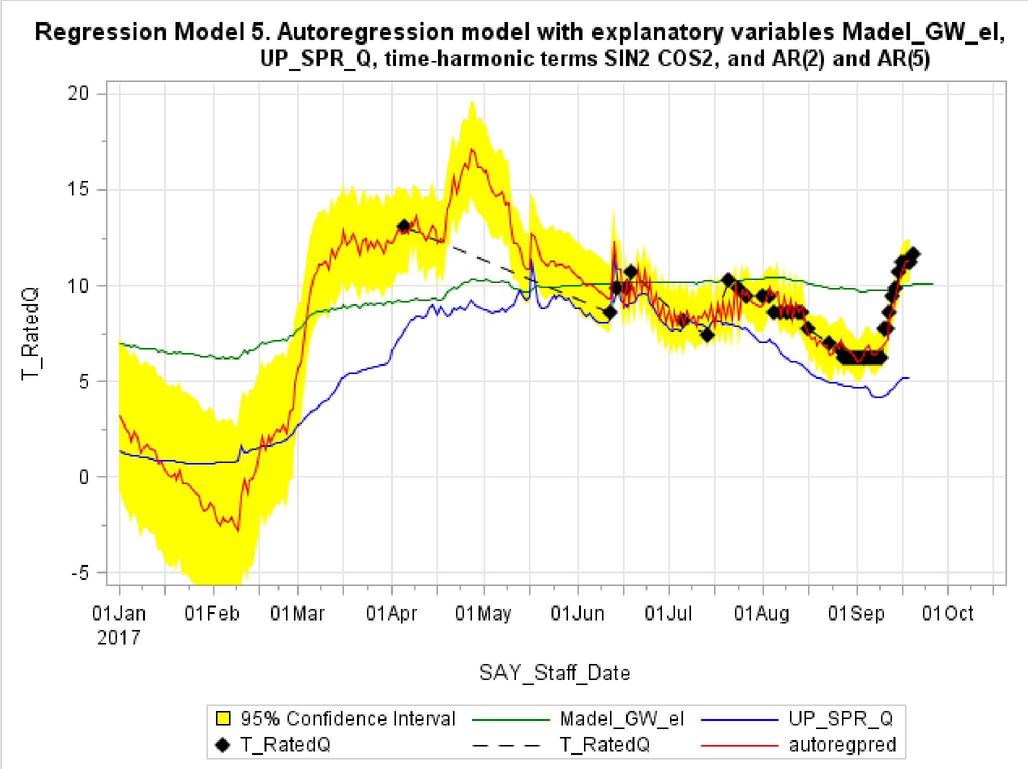
NOTE: Pr<DW is the p-value for testing positive autocorrelation, and Pr>DW is the p-value for testing negative autocorrelation.

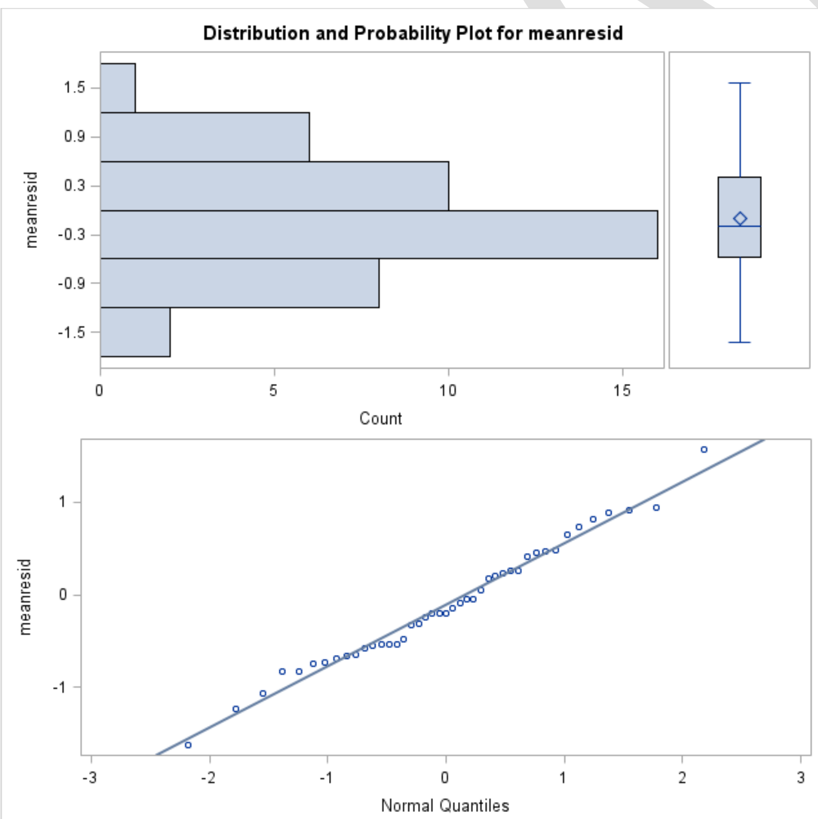
Regression Model 5. Autoregression model with explanatory variables Madel_GW_el, UP_SPR_Q, time-harmonic terms SIN2 COS2, and AR(2) and AR(5)

Autoregressive parameters assumed given

Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|	Variable Label
Intercept	1	-47.0395	9.0091	-5.22	<.0001	
Madel_GW_el	1	5.7559	1.0354	5.56	<.0001	
SIN2	1	1.6413	0.7003	2.34	0.0247	Sine*Theta
COS2	1	9.0703	0.9436	9.61	<.0001	Cosine*Theta
UP_SPR_Q	1	0.7589	0.2183	3.48	0.0013	




Regression Model 5. Autoregression model with explanatory variables Madel_GW_el, UP_SPR_Q, time-harmonic terms SIN2 COS2, and AR(2) and AR(5)
T_RatedQ
SAY_Staff_Date
01Jan 2017
01Feb
01Mar
01Apr
01May
01Jun
01Jul
01Aug
01Sep
01Oct
95% Confidence Interval
Madel_GW_el
UP_SPR_Q
T_RatedQ
T_RatedQ
autoregpred


Distribution and Probability Plot for meanresid
meanresid
Count
meanresid
Normal Quantiles

Regression Model 5. Autoregression model with explanatory variables Madel_GW_el, UP_SPR_Q, time-harmonic terms SIN2 COS2, and AR(2) and AR(5)

The UNIVARIATE Procedure
Variable: meanresid

Moments

N	43	**Sum Weights**	43
Mean	-0.1066324	**Sum Observations**	-4.5851921
Std Deviation	0.6623035	**Variance**	0.43864593
Skewness	0.21519115	**Kurtosis**	-0.0527954
Uncorrected SS	18.9120591	**Corrected SS**	18.4231292
Coeff Variation	-621.10922	**Std Error Mean**	0.10100034

Basic Statistical Measures

Location		**Variability**	
Mean	-0.10663	**Std Deviation**	0.66230
Median	-0.19706	**Variance**	0.43865
Mode	.	**Range**	3.19019
		Interquartile Range	0.98860

Tests for Location: Mu0=0

Test	**Statistic**		**p Value**	
Student's t	**t**	-1.05576	**Pr > |t|**	0.2971
Sign	**M**	-4.5	**Pr >= |M|**	0.2221
Signed Rank	**S**	-93	**Pr >= |S|**	0.2664

Tests for Normality

Test	**Statistic**		**p Value**	
Shapiro-Wilk	**W**	0.986434	**Pr < W**	0.8848
Kolmogorov-Smirnov	**D**	0.089404	**Pr > D**	>0.1500
Cramer-von Mises	**W-Sq**	0.044449	**Pr > W-Sq**	>0.2500
Anderson-Darling	**A-Sq**	0.276507	**Pr > A-Sq**	>0.2500